Lec 5; 09/02/2009

Review of Classical Mechanics;

Particles;

In classical mechanics, a particle obeys Newton's 2nd law;

$$\vec{F} = m\vec{a} \xRightarrow[\text{forces}]{\text{Conservative}} m\frac{d^2\vec{r}}{dt^2} = -\nabla \underbrace{V(\vec{r})}_{\text{potential}}$$

In one dimension;

$$m\frac{d^2x}{dt^2} = -\frac{dV(x)}{dx}$$

For a given potential, $x(t)$ is known if $x(0)$, $\dot{x}(0)$ are specified (initial Conditions). $\cdot \equiv \frac{d}{dt}$

Equation of motion can be found from Lagrangian formulation;

$L = T - V$ $\qquad$ T: kinectic energy $= \frac{1}{2}m\dot{x}^2$

Euler-Lagrange equation;

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

This yields;

$$m\ddot{x} + \frac{dV(x)}{dx} = 0$$

Momentum p is defined as;

$$p \equiv \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

Hamiltonian H is found from Lagrangian by a Legendre transformation;

$$H = \dot{x}p - L = T + V$$

One obtains Hamilton's canonical equations;

$$\frac{\partial H}{\partial p} = \dot{x} \quad , \quad \frac{\partial H}{\partial x} = -\dot{p}$$

Now we have two first-order differential equations instead of one second-order equation. For a given V, we can find $x(t)$, $p(t)$ provided that $x(0)$, $p(0)$ are known.

Example: Harmonic oscillator $V = \frac{1}{2}kx^2$.

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} \quad , \quad \frac{\partial L}{\partial x} = -kx$$

Euler-Lagrange equation then gives:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = m\ddot{x} + kx = 0$$

Hamiltonian H is given by:

$$H = \dot{x}p - L \qquad p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

$$H = m\dot{x}^2 - \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

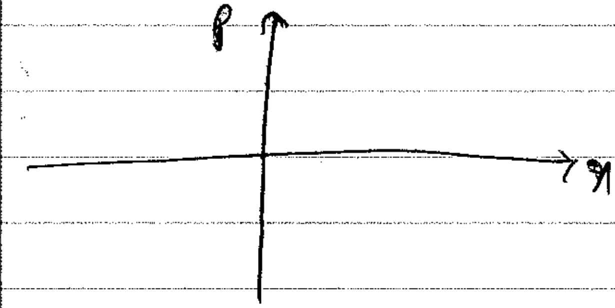
$$\frac{\partial H}{\partial p} = \frac{p}{m} = \dot{x}$$

$$\frac{\partial H}{\partial x} = kx = -\dot{p} = -m\ddot{x} \Rightarrow m\ddot{x} + kx = 0$$

Note that in classical mechanics position x and momentum p of a particle can be known to arbitrary precision at the same time. Once we know x, p at a given time, then we know x, p at all other times.

The state of a particle is uniquely specified by $x(t), p(t)$. The two-dimensional space

x, p is called the phase space;



If we know x, p at $t=0$, then we can find x, p at any time using Hamilton's equations. The motion of the particle is given by a trajectory in the phase space. Only one trajectory passes through a given point in the phase space.

Example: Harmonic oscillator $V = \frac{1}{2} k x^2$.

It is easy to see that the energy is conserved.

Thus:

$$\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 = \text{Const.} \Rightarrow \frac{p^2}{2m} + \frac{1}{2} k x^2 = \text{Const.}$$

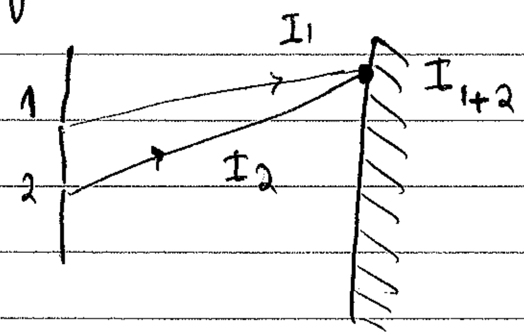
This represents an ellipse in the phase space.

Trajectory is closed because the motion is periodic.

carry energy $E = hf = \hbar\omega$, where h is the Planck's constant ($h = 6.63 \times 10^{-34}$ Js).

Photons can be considered as particles, but not those that we know clasically. If it were the case, i.e. if light (electromagnetic waves) were a collection of classical particles, there would be no interference or diffraction.

To illustrate this, consider the double-slit experiment:



Here I_1 is the intensity at a point on the screen if only slit 1 is open (similarly for I_2)

I_{1+2} is the intensity when both slits are

Waves:

In classical physics we also have extended objects (string) or continuous quantities (electromagnetic field). These are not localized like particles. Equation of motion is typically wave equation. The wave nature leads to phenomena like interference and diffraction that do not exist for a particle. There is therefore a clear distinction between particles and waves in classical physics.

However, wavelike and particle-like behaviors are simultaneously observed in some cases. To explain the photoelectric effect, Einstein suggested that light consists of quanta (called photon) that open. For a collection of classical particles we have $I_{1+2} = I_1 + I_2$. While, for a wave this is not the case.

It was suggested by de Broglie that, just as waves can have particle-like behavior, particles can also have wave-like behavior. In analogy with photons, the wavelength associated with a particle with momentum p is given by:

$$\lambda = \frac{h}{p} \quad \text{(de Broglie wavelength)}$$

In this picture we have a wave-particle duality.

The question is how to describe particles if they exhibit wave-like behavior (and vice-versa

Here comes the probability interpretation.

A wave $\Psi_{(x)}$ is associated with a particle that acts as a pilot. $|\Psi_{(x)}|^2$ gives the probability of finding particle at x.

Note that in this picture the position and momentum of a particle cannot be known simultaneously. For example, photons with a definite momentum correspond to a monochromatic wave that has infinite extension, imlying that a photon can be found anywhere.